Stats 2MB3, Tutorial 9

Mar 20th, 2015

Prediction Interval

- A prediction interval (PI) for a single observation to be selected from a normal distribution is

$$\bar{x} \pm t_{\alpha/2,n-1} \cdot s\sqrt{1+\frac{1}{n}}$$

- The prediction level is 100(1-α)%. A lower prediction bound results from replacing $t_{\alpha/2}$ by t_{α} and discarding the + part and a similar modification gives an upper prediction bound.

Confidence Interval for Variance

- A 100(1-α)% confidence interval for the variance σ^2 of a normal population is given by

$$\left(\frac{(n-1)s^2}{\chi^2_{\alpha/2,n-1}}, \frac{(n-1)s^2}{\chi^2_{1-\alpha/2,n-1}}\right)$$

- The CI for σ has lower and upper limits that are square roots of the limits above. An upper or lower confidence bound results from replacing α/2 with α in the corresponding limit of the CI.

Exercise 29, page 292

- Determine the t critical values that will capture the desired t-curve area in each of the following cases:
- 1. Central area=0.95, df=10;
- 2. Central area=0.95, df=20;
- 3. Central area=0.99, df=20;
- 4. Central area=0.99, df=50;
- 5. Upper-tail area=0.01, df=25;
- 6. Lower-tail area=0.025, df=5

- By R:
- 1. qt(1-(1-0.95)/2, 10)=2.228139
- 2. qt(1-(1-0.95)/2,20)=2.085963
- 3. qt(1-(1-0.99)/2,20)=2.84534
- 4. qt(1-(1-0.99)/2, 50)=2.677793
- 5. qt(1-0.01, 25)=2.485107
- 6. qt(0.025, 5)=-2.570582

Exercise 37, page 293

- A study of ability of individuals to walk in a straight line reported the accompanying data on cadence (strides per second) for a sample of n=20 randomly selected healthy men.
- 0.95,0.85,0.92,0.93,0.86,1.00,0.92,0.85,0.81,0.78 ,0.93,0.93,1.05,0.93,1.06,1.06,0.96,0.81,0.96
- A normal probability plot gives substantial support to the assumption that the population distribution of cadence is approximately normal. A descriptive summary of the data from follow:

Exercise 37, page 293

- Sample size: 20
- Mean: 0.9255
- Median: 0.9300
- Trimmed Mean: 0.9261
- Standard Deviation: 0.0809
- Standard Error of Mean: 0.0181
- Minimum: 0.7800
- Maximum: 1.0600
- Lower Quantile: 0.8525
- Upper Quantile: 0.9600

- a) Calculate and interpret a 95% confidence interval for population mean cadence;

- b) Calculate and interpret a 95% prediction interval for the cadence of a single individual randomly selected from this population;

- a)
- $t_{0.025}$ (20-1)=2.093024
- The 95% confidence interval (0.9255-2.093*0.0181, 0.9255+2.093*0.0181)=(0.8876, 0.9634).

- b)
- The 95% prediction interval

$$0.9255 \pm 2.093(0.0809)\sqrt{1+\frac{1}{20}} = (17.25, 32.75)$$

Exercise 45, page 296

- The following observations were made on fracture toughness of a base plate of 18% nickel maraging steel:
- 69.5,71.9,72.6,73.1,73.3,73.5,75.5,75.7,75.8,76.1,76.2,76.2,77.0,77.9,78.1,79.6,79.7,79.9,80.1,82.2,83.7,93.7
- Calculate a 99% CI for the standard deviation of the fracture-toughness distribution. Is this interval valid whatever the nature of the distribution? Explain.

- Degree of freedom: n-1=22-1=21
- qchisq(0.005,21)=8.033653
- qchisq(0.995,21)=41.40106
- The variance s^2 =25.36799
- Then the CI for σ^2 is

(21*25.368/41.401,21*25.368/8.034)

= (12.868, 66.309)

CI for σ is (3.6, 8.1)

This interval is valid when the distribution is approximately normal. In other words, the observations must pass the normality test.